

# Wavelet Theory-Based Analysis of High-Frequency, High-Resolution Electrocardiograms: A New Concept for Clinical Uses

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## Summary

Recently, the wavelet transform has been considered as an alternative to the Fourier transform in many fields that require frequency analysis, signal processing, or image processing. Unlike the Fourier transform, the wavelet transform is two-dimensional in time and frequency, and allows data in both domains to be analyzed at the same time. Because the wavelet transform is not able to exhibit its superlative resolving power when used with ordinary electrocardiographs, we acquired electrocardiogram waveforms at sampling rates of 5, 10, and 20 kHz. According to the analysis of the adult electrocardiogram (ECG) presented in this paper, the QRS complex under the continuous wavelet transform has a gourd shape. The P wave, which can be recognized prior to the gourd shape as knots, was disintegrated into stratified structural components that are thought to correspond to the impulse-conduction system. Potentials thought to derive from the sinus node and His fasciculae were observed too. Since we are able to observe the atrial conduction system noninvasively, we think that this technique may be effective in the diagnosis and treatment of arrhythmias. The analysis of the ventricular conduction system shows that the T wave consists of four main stratified components, T1, T2, T3, and T4. The main structural component T4 has an elongated spindle shape along the time axis at frequencies below the gourd shape of the QRS complex. The endpoint of the T wave can be defined mathematically by determining the point where T3 and T4 fuse. We think this analysis will be effective for evaluating the risk of heart disease and the effectiveness of antiarrhythmic drugs.

## Key Words

Wavelet transforms, impulse-conduction system, QT interval, morphological characteristics of ECGs

## Introduction

Recently, the wavelet transform has been considered as an alternative to the Fourier transform in many fields that require frequency analysis, signal processing, or image processing. Unlike the Fourier transform, the wavelet transform is two-dimensional in time and frequency, and allows data in both domains to be analyzed at the same time. The wavelet transform has been described as a mathematical microscope for the natural sciences [1], and is an extremely powerful technique

for grasping subtle changes and discontinuities in surface ECG recordings. The sampling rate of earlier ECGs was 250 Hz, and even signal averaging ECGs only use the low rate of 1 kHz. Since the sampling rate is low and only a few multiresolution analyses can be performed at the 250 Hz, the wavelet transform is not able to exhibit its superlative resolving power when used with ordinary ECGs. Therefore, sampling rates of 5 to 20 kHz are needed, which are appropriate to allow

wavelet transform analysis to exhibit its abilities. Current ECG research has no scientific method for determining the starting points and endpoints of the P wave, QRS complex, and T wave. However, individual doctors determine these points either intuitively or arbitrarily, and the process has not been standardized. It is known, however, that variations in the QT interval, i.e., QT dispersion, indicates an unevenness in the local depolarization process in each ventricle. Increases in QT dispersion are related to a risk of ventricular tachycardia and sudden cardiac death, and the QT dispersion is often used in risk evaluation of acute myocardial infarction, congestive heart failure, dilated cardiomyopathy, and in the evaluation of the effectiveness of antiarrhythmic medication [2]. By using a high-frequency high-resolution ECG (HFHR-ECG) we investigated the possibilities of determining the starting points and endpoints of the P wave, QRS complex, and T wave.

## Materials and Methods

### Wavelet Theory

Wavelet theory is designed to give good time resolution and poor frequency resolution at high frequencies, and good frequency resolution and poor time resolution at low frequencies. This approach is useful for ECG signals, i.e., signals with high-frequency components for short durations, and low-frequency components for long durations. From a one-dimensional input signal  $f(t)$ , in this case the ECG signal, the continuous wavelet transformation is a two-dimensional function

$$CWT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \Psi^* \left( \frac{t-b}{a} \right) dt$$

of a scale parameter ( $a \sim 1/\text{frequency} > 0$ ) and a translation parameter ( $b = \text{time localization at which the signal is analyzed}$ ). There are several wavelet functions (mother wavelets  $\mathbf{Y}(t)$  with  $\mathbf{Y}^*(t)$  conjugate complex) available with different properties. For example, the Morlet wavelet [3,4] and Gabor-8-Power wavelet [5] are excellent for achieving high frequency resolution. On the other hand, the MexicanHat wavelet has a poor frequency resolution but a good time resolution because there is little distortion towards the time axis, and thus, there is excellent linearity [6]. Consequently, we analyzed the morphological characteristics of ECG waveforms in the Gabor-8-Power continuous wavelet

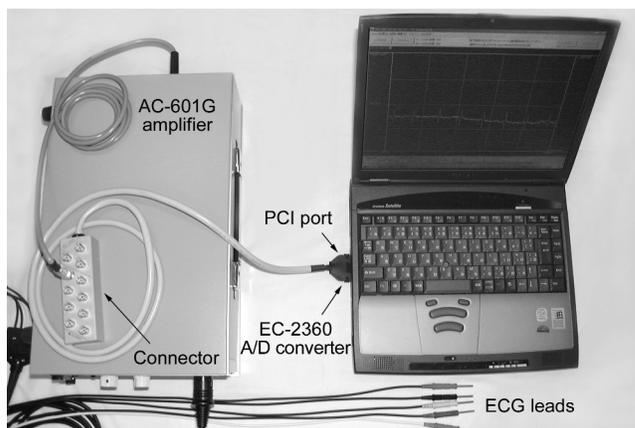


Figure 1. The high-frequency, high-resolution ECG (HFHR-ECG). The raw signal from the ECG leads is amplified by the AC-601G ECG amplifier (Nihon Kohden, Japan) and digitalized by the EC-2360 A/D converter (Elmec, Japan), which is connected to the notebook personal computer as a PCI card.

transform (CWT), while checking the position of the P wave, QRS, and T wave in the MexicanHat CWT.

Some other methods of wavelet analysis were used to check the results. The method of Daubechies [7] was used for discrete wavelet transforms and the Wickerhauser methods [8] were used for wavelet packet transforms with the best basis algorithm. All of these analyses were performed using the MEM software (Wavelet Analysis and Spectrum Analysis Software [9]), which provides both color charts for presentations and gray scale charts for journal articles.

### High-Frequency, High-Resolution ECG

We developed a high-frequency, high-resolution ECG (HFHR-ECG, see Figure 1). The ECG data (lead II measurement of the cardiac electromotive force vector) is obtained as time series data using an AC-601G ECG amplifier (Nihon Kohden, Japan). This data is digitized with an EC-2360 A/D converter (Elmec, Japan) acquired using a notebook personal computer, and saved as a data file. Sampling rates from 5 to 20 kHz are used.

### Patients

The standard surface ECG of a healthy, 38-year-old male, and of a 55-year-old male suffering from chest pain, and treated with nifedipine and dipyridamole were analyzed using CWT.

## Results

### Normal ECG

Some typical CWTs from an adult ECG waveform are presented. In each CWT image, the vertical axis represents frequencies with lower frequencies at lower positions, and the horizontal axis represents time. In the gray scale images, the more white the image, the higher the power. The upper panel in each figure shows the raw ECG waveform. Examination of the position of the P wave, the QRS complex, and the T wave was performed using the MexicanHat CWT (Figure 2a), whereas excellent frequency resolution is presented in the Gabor-8-Power CWT (Figures 2b and 2c).

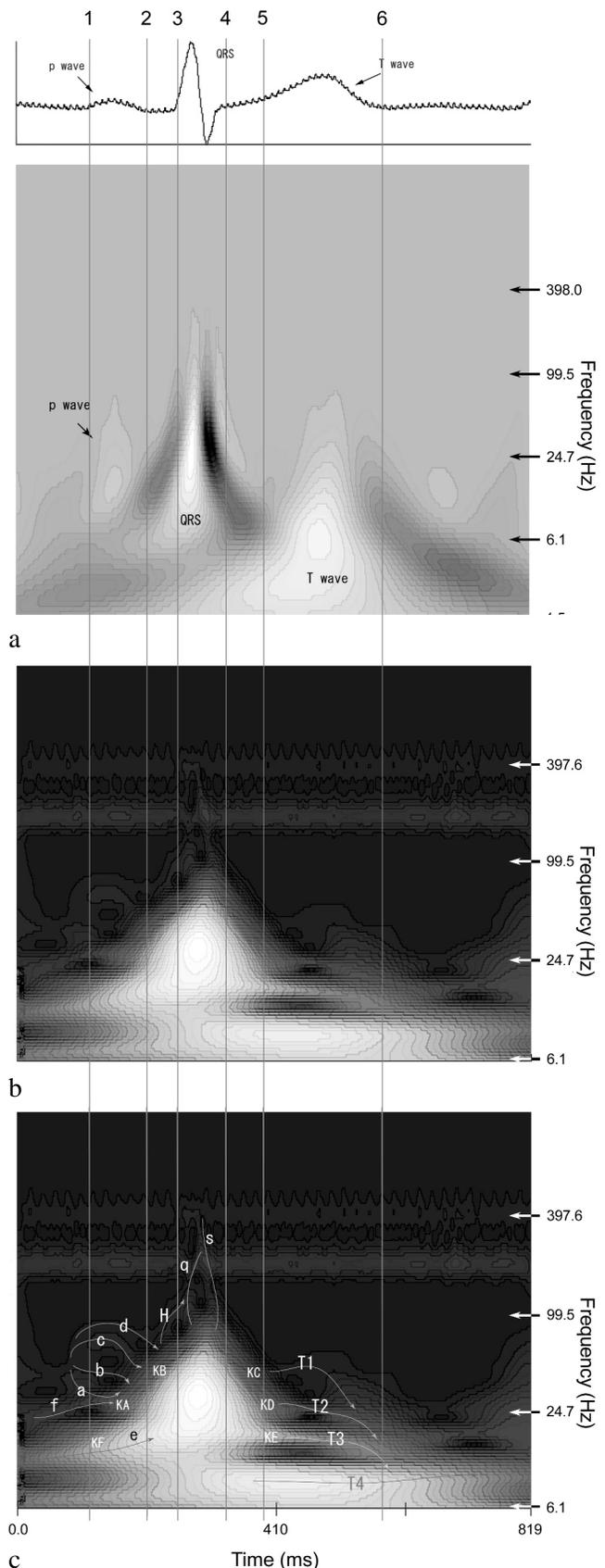
While a "gourd" type or "pear" type QRS complex with rather high power (indicated in white) can be seen to the lower left of center, the main structural component of the T wave, which is a horizontally elongated spindle shape, can be seen to the lower right of that feature. In the relationship between the main structural component of this T wave and QRS, the pathological form differs, as will be described later, in the flattening and lowering of the T wave and in changes in the ST components. Furthermore, an extension of the structural component of the P wave can be recognized at the left end of the gourd type of QRS. Several halo-like lines can be seen around the QRS, covering it like a fog. The lines a, b, and c in Figure 2c are the three conduction pathways in the right atrium, and their startup areas are the same. These startup areas are thought to be the sinus node, which could not be measured with

*Figure 2. Continuous wavelet transform (CWT, gray scale images) of a high-frequency high-resolution ECG (upper curve, ECG lead II, sampling rate = 5 kHz, heart rate = 84 beats/min).*

*Panel a) The ECG waveform in the MexicanHat CWT, which shows good time resolution but poor frequency resolution. The position of the P wave, QRS complex, and T wave in the CWT is checked by comparing the CWT and raw ECG waveform (vertical lines 1-6).*

*Panel b) Structural components of the P wave and T wave and typical waveform components in the Gabor-8-Power CWT.*

*Panel c) CWT of Panel b with additional information. Knots of the P wave (KF, KA, KB); knots of the T wave (KC, KD, KE); knot of a superimposed low-frequency part of the P wave and the QRS complex (KF); lines of the right atrial conduction pathway (a, b, c); left atrial conduction pathway (d); other components of the P wave (e, f); His bundle potential (H), QRS complex (q, s); and components of the T wave (T1-T4).*



conventional ECGs. The lines a, b, and c finally fuse to form only one line, and are conducted to the atrioventricular node. Line d is thought to be related to the Bachmann's bundle, which is the conduction pathway within the left atrium. Line H corresponds to the His bundle potential. In addition, the P wave is also thought to consist of the e, f, and other waveforms. While the T wave consists of T1, T2, T3, T4, and other components, these finally fuse – in order corresponding with the ECG waveform. However, when there is a rise or fall in the ST position, differences appear in the height on the frequency axis of T3 and f, which is a structural component of the P wave, or T3 and T4. Additionally, if there is a flattening and lowering of the T wave, T1, T2, and T3 become level and disappear with almost no fusion. The space between q and s corresponds to the QRS complex in a conventional ECG, and normally this forms a line as shown in Figure 2. However, if myocardial damage is present, it appears as though several small circuits are formed between q and s, a scenario which can lead to arrhythmias [10]. Figure 2 shows hypothetical attempts at determining the starting points and endpoints of the P wave, QRS complex, and T wave in the CWT of an ECG waveform. Line 1 is the starting point of the P wave, and if a knot KA is present in the CWT, the P wave starts in the vicinity of the left edge of that knot (the "knot" here is the feature that looks like a bump, of which we can see several within the QRS complex). The origin of the sinus node potential is thought to be the starting point of lines a,b,c, and d. Lines a, b, and c reflect the three conduction systems in the right atrium, and line d reflects the left atrial conduction system, the Bachmann's bundle. Line 2 passes through the endpoint of the P wave, close to knot KB. Line H is thought to be the His bundle potential. Line 3 passes through the starting point of the QRS complex, corresponding to the endpoint of line H, and also the starting point of line q. Line q is at the left side of the section that rises from the outline of the QRS complex itself. Line 4 passes through the endpoint of the QRS complex. Line s fuses with the outline of the QRS complex itself at the left side of line 4. Line 5 passes through the starting point of the T wave, and also through the starting point of T1 or KC. Line 6 passes through the end point of the T wave where T3 and T4 fuse, or the vicinity of the right side of KE. The above hypothesis allows us to determine these points from the CWT figure, even if the P wave and T wave end-

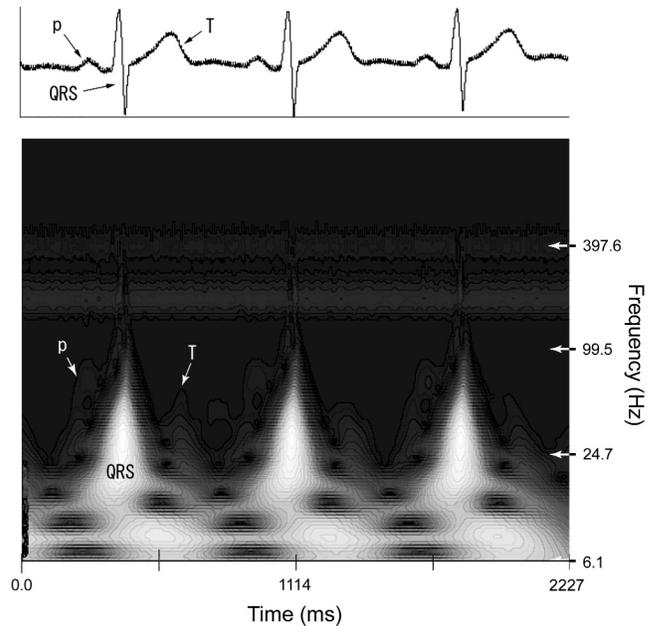


Figure 3. Continuous wavelet transformation (CWT, gray scale images) of a high-frequency, high-resolution ECG (upper curve, ECG lead II, sampling rate = 5 kHz, heart rate = 84 beats/min). The three consecutive ECG waveforms are very similar to Figure 2b, which shows another time interval in the same ECG, down to the fine structures including the halos.

points are unclear in the original ECG waveform. Figure 3 shows the CWT of the same HFHR-ECG of Figure 2 for another time interval. The three consecutive ECG waveforms are almost identical to Figure 2b, down to the fine structures including the halos.

#### Abnormal ECG

Figure 4 shows the CWT of a 55-year-old male with myocardial damage. The upper section shows the conventional ECG, in which the P wave is unclear and the T wave has become flattened and lowered due to myocardial damage. The starting points and endpoints of the P wave and T wave cannot be determined from the conventional ECG. However, the displacement to the low frequency side of the e and f lines, which are structural components of the P wave, can be seen in the CWT image. The starting point of the P wave is the left edge of KA (the black upward-pointing arrows in Figure 4). The endpoint of the P wave is the base of KB. The starting point of the T wave, KC, and T1, T2, and T3, are short and remain essentially horizontal until they either fade away or fuse slightly (the white downward-pointing arrows in Figure 4). However, the power of that sig-

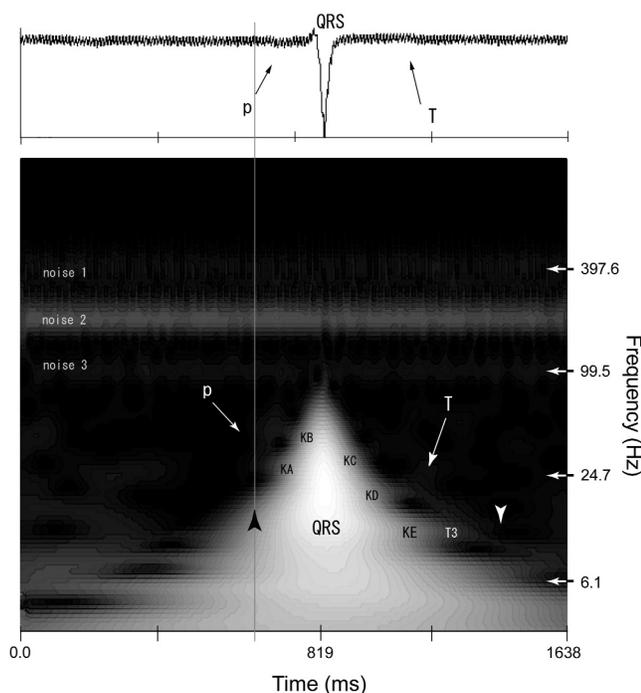


Figure 4. Continuous wavelet transformation (gray scale images) of a high-frequency, high-resolution ECG (upper curve, ECG lead  $V_1$  of a 55-year-old male with myocardial damage, sampling rate = 5 kHz, heart rate = 60 beats/min). The starting point and endpoint of the P wave and T wave are not clear in the conventional ECG. While T1, T2, and T3 (characteristic waveform components of the T wave) are short and only fuse slightly, their power is extremely weak compared to a normal ECG. If a vertical line is drawn up from the left edge of KA (characteristic knots KA, KB, KC, KD, KE), the intersection point with the original ECG waveform is the starting position of the P wave.

nal is extremely weak compared to normal. When these waves do not join, the right end of T3 is thought to be the endpoint of the T wave. The intersection of a vertical line positioned at the right edge of KA, which is somewhat above the center of the black arrow, with the original ECG is the starting point of the P wave. The intersection of a vertical line extended from the point indicated by the white arrow, where either T3 fades away or T3 and T4 fuse slightly, and the original ECG at the top of the figure is the endpoint of the T wave.

## Discussion

### Normal ECG

We described phenomena in which the conduction systems of the heart, e.g. in the right atrium (lines a, b, and c in Figure 2c) and in the ventricles (lines T1, T2, T3, and

T4 in Figure 2c), were observed in the CWT of standard surface ECG waveforms. This phenomenon can only be observed at sampling rates of 5 kHz and higher. Wavelets functions such as Gabor-8-Power are needed that have good frequency resolution in the vicinity of the P wave and T wave, whereas these effects cannot be observed with genuine wavelets, e.g., MexicanHat, which have poor frequency resolution.

By ECG analysis, changes in the electrical vector (the time series data for the electromotive force vector) are observed for the three dimensions of the heart at a single point in two dimensions. When observed in this way, the frequency increases for vector changes towards the point of observation, and decreases for changes moving away from the point of observation. Therefore, in the CWT, if the starting points and endpoints of the impulse-conduction system are determined, we expect that it should be possible to observe changes in power describing upward or downward arcs. Since the sampling rate is so low in conventional ECGs, the P wave appears as a single wave. However, as we already know physiologically, these waves pass through four pathways, and the stimulus is transmitted to the ventricle through the His bundle as a single unit. Our method with CWT of a HFHR-ECG allows this to be observed as a Doppler effect.

### Abnormal ECG

Even if the starting points and endpoints of the P wave and T wave are unclear, in arrhythmias the morphological features of the ECG waveform on the CWT, such as the KA and KB knots and lines a, b, and c, can be used to extend vertical lines to the original ECG at the top of the figure and determine the intersection points, and thus infer the starting points and endpoints. Wavelet packet analysis is performed using the best basis algorithm with information entropy (amount of uncertainty) as the cost function. We obtained the lowest cost (i.e., best fit) with the  $w_{7,0}$  crystal (7th order approximation function of the raw ECG signal in multiresolution analysis) with Daubechies20 and Coiflet24 multiresolution analysis [9,11]. We analyzed this  $w_{7,0}$  crystal by Gabor-8-Power CWT and obtained an image very similar to Figure 2b.

## Conclusion

Using the HFHR-ECG for CWT, one can determine the starting points and endpoints of the P wave, QRS

complex, and T wave mathematically. We think that detection of the conduction system of the P wave by HFHR-ECG is useful for the diagnosis and treatment of arrhythmias. Indeed, it may complement the cardiac catheterization method as a diagnostic procedure. By defining the point where two of the T wave components, T3 and T4, fuse as the endpoint of the T wave, we can determine the QT interval, which has not been measured mathematically. We think HFHR-ECG will be effective for evaluating the risk of heart disease and the effectiveness of antiarrhythmic drugs.

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